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13. ABSTRACT *(Maximum 200 words)*

The project aims to explore through a variety of implementations a paradigm for the analysis of scientific and operational phenomena that depend on formalizations of uncertainty through probabilistic models. Such models can be interpreted as representations of random processes, or they can be interpreted as representations of the uncertainty of an analyst facing a situation described by the model. These interpretations are often viewed as mutually exclusive, but we regard them as complementary, and hence simultaneously applicable. A major motivating reason for constructing the models is to facilitate making uncertain inferences, followed in many cases by decision making informed by the inferences.

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The central principle of probabilistic inference remains, as it has been for 200 years, the Bayesian principle of updating inferences by formal computation of conditional probabilities, that is, by conditioning on the stream of incoming data. The belief function principle is a relaxation of the Bayesian rule, first suggested in special cases by R. A. Fisher about 65 years ago under the name fiducial inference, that retains the feature of conditioning on the data but does not require the full specification of a priori probabilities for all eventualities represented by the model. For example, in the ubiquitous class of Gaussian linear models the Bayesian formulation relies on awkward "improper priors" that are artifacts, that is, do not specify meaningful uncertainty judgments, whereas the normal linear belief function model dispenses with such priors and proceeds directly to conditioning. There are debates over "axioms" and principles that bear on the acceptability of belief function arguments, as compared with the more purist Bayesian models that have a counterbalancing drawback that they demand more in the way of prior knowledge than the analyst may wish to formalize in probability terms.

MODELLING PROBABILISTIC AND LOGICAL RELATIONS WITH BELIEF FUNCTIONS

FINAL REPORT

ARTHUR P. DEMPSTER

JUNE 1995

U. S. ARMY RESEARCH OFFICE

GRANT NO. DAAL03-91-G-0089

DEPARTMENT OF STATISTICS, HARVARD UNIVERSITY

STATEMENT OF PROBLEM STUDIED

The goal of the project was to build on work in the 80's by the PI and coworkers including Glenn Shafer and Augustine Kong on the development of theory, especially computing strategies and algorithms, for the application of belief functions on networks to assessing posterior uncertainty about complex systems.

SUMMARY OF IMPORTANT RESULTS

1. The thesis S-158 showed the applicability of the theory of normal belief functions to the analysis of a complex pulsatile phenomenon. The overall Bayesian model was computed using MCMC (Markov Chain Monte Carlo) that assessed posteriors on the number, location, and amplitude of pulses. One paper, S-167, is accepted for publication, and others are in preparation.

2. A related problem of locating signals in space that are contaminated with noise has been addressed, as summarized in S-169. Assessing the posterior in this case led by serendipity to a very general scheme of weighted sampling that has many other practical applications, as described in the thesis S-168. Other technical reports in this series are S-159 (published in *Biometrika*), S-164 (submitted) and S-165. Further reports and submissions are planned.

3. Problems of diagnosis and prognosis have been opened up, resulting in published commentary (S-156 and S-163). This has led to a developing project to analyze the management of intensive care units at Massachusetts General Hospital.

REPORTS AND PUBLICATIONS

S-156 "Comment: Assessing the Science Behind Graphical Modelling Techniques." A.P. Dempster, *Statistical Science* **8**, 247-250, 1993 (will be sent shortly).

S-158 "A Bayesian Analysis of Diurnal Cortisol Series." Patricia M. Meehan. (Ph.D. thesis June 1993, transmitted in 1994).

S-159 "Weighted Finite Population Sampling to Maximize Entropy." Xianghui Chen, A.P. Dempster, and Jun S. Liu, *Biometrika* **81**, 457-469, 1994 (will be sent shortly).

S-163 "Models and Modelling in Context." A.P. Dempster and E.N. Brown, Ch 3 of *Probabilistic Reasoning and Bayesian Belief Networks* (ed. A. Gammerman) Alfred Waller Limited, Publishers 1995 (will be sent shortly).

S-164 "Computations of Poisson-Binomial Probabilities in Connection with Weighted Sampling and Retrospective Studies." Xianghui Chen and Jun S. Liu, 1994 (manuscript will be sent shortly).

S-165 "Poisson-Binomial Distribution, Conditional Bernoulli Distribution, Entropy and Mutual Dependence." Xianghui Chen, 1995.

S-167 "A Belief Function Approach to Likelihood Updating in a Gaussian Linear Model." P.M. Meehan, A.P. Dempster, E.N. Brown. To appear in *Bayesian Statistics 5*, Oxford University Press 1995 (manuscript will be sent shortly).

S-168 "Weighted Sampling, Statistical Applications and Generalizations." Xianghui Chen, Ph.D. thesis, 1995 (will be sent shortly).

S-169 "Metropolis Algorithm and the Nearly Black Object." Xianghui Chen, 1995.

STUDENTS SUPPORTED

Patricia Meehan (Ph.D. 1993), Xianghui Chen (Ph.D. 1995), Igor Perisic (Ph.D. candidate)